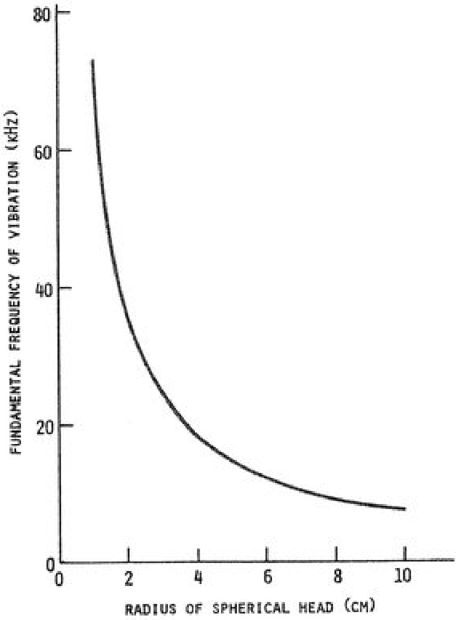

Fig. 4. The fundamental frequencies of sound generated inside the head as a function of spherical head radii.

The fundamental frequency of sound generated inside the spherical head is therefore given by

$$f_1 = c_1/2a. \tag{32}$$

Fig. 4 is a plot of the fundamental frequency of sound generated in the head as a function of head radii. The frequency varies from above 80 kHz for mice ($a \cong 1$ cm) to about 8 kHz for humans ($a = 7$–10 cm).

We evaluate the constants A_m by using the initial conditions in (14) to obtain

$$A_m = -u_0 \left\{ \frac{a}{N\pi} \int_0^a r^2 j_1(k_m r) j_1\left(\frac{N\pi r}{a}\right) dr \pm \frac{4\mu}{3\lambda + 2\mu}\left(\frac{1}{N\pi}\right)^2 \int_0^a r^3 j_1(k_m r)\, dr \right\} \Bigg/ \left\{ \int_0^a r^2 [j_1(k_m r)]^2\, dr \right\}, \quad N = \begin{cases} 1,3,5,\cdots \\ 2,4,6,\cdots \end{cases} \tag{33}$$

The integrals in (33) may be evaluated [24] to give

$$\int_0^a r^2 j_1(k_m r) j_1\left(\frac{N\pi r}{a}\right) dr = \left[\frac{-a^3}{(k_m a)^2 - (N\pi)^2}\right]\left[\pm \frac{ka}{N\pi}\right] j_0(k_m a), \quad N = \begin{cases} 1,3,5,\cdots \\ 2,4,6,\cdots \end{cases} \tag{34}$$

$$\int_0^a r^3 j_1(k_m r)\, dr = \left(\frac{a^2}{k_m a}\right)^2 [3 j_1(k_m a) - k_m a j_0(k_m a)] = \frac{a^3}{k_m} j_2(k_m a) \tag{35}$$

$$\int_0^a r^2 [j_1(k_m r)]^2\, dr = \frac{a^3}{2}\{[j_1(k_m a)]^2 - j_0(k_m a) j_2(k_m a)\} \tag{36}$$

where $j_2(k_m a)$ is the spherical Bessel function of the first kind and second order.

Using these values (33) becomes

$$A_m = \mp u_0 a \left(\frac{1}{N\pi}\right)^2 \left[\frac{2}{[j_1(k_m a)]^2 - j_0(k_m a) j_2(k_m a)}\right] \cdot \left\{ \frac{4\mu}{3\lambda + 2\mu}\left(\frac{1}{k_m a}\right) j_2(k_m a) - k_m a j_0(k_m a) \frac{1}{(k_m a)^2 - (N\pi)^2} \right\}, \quad N = \begin{cases} 1,3,5,\cdots \\ 2,4,6,\cdots \end{cases} \tag{37}$$

For $k_m a = m\pi = N\pi$, (37) simplifies to

$$A_m = -u_0 a \left(\frac{1}{N\pi}\right)\left[1 + \frac{24\mu}{3\lambda + 2\mu}\left(\frac{1}{N\pi}\right)^2\right]. \tag{38}$$

The displacement response of the sphere to a step input of microwave energy is now given by introducing (37) in (30) and then combining (24) and (30) in (15). We have

$$u(r,t) = u_0 Q + \sum_{m=1}^{\infty} A_m j_1(k_m r) \cos \omega_m t \tag{39}$$

$$Q = \frac{a}{N\pi} j_1\left(\frac{N\pi r}{a}\right) \pm \frac{4\mu}{3\lambda + 2\mu} \frac{r}{N^2\pi^2}, \quad N = \begin{cases} 1,3,5,\cdots \\ 2,4,6,\cdots \end{cases} \tag{40}$$

The radial stress can be deduced from the displacement solution using [22] and (13):

$$\sigma_r(r,t) = (\lambda + 2\mu)\frac{\partial u}{\partial r} + 2\lambda \frac{u}{r} - \beta v. \tag{41}$$

We have, therefore, by substituting (6), (10), and (39) into (41),

$$\sigma_r(r,t) = 4\mu u_0 S + \sum_{m=1}^{\infty} A_m k_m M_m \cos \omega_m t \tag{42}$$

$$S = \pm \left(\frac{1}{N\pi}\right)^2 - j_1\left(\frac{N\pi r}{a}\right) \Bigg/ \left(\frac{N\pi r}{a}\right), \quad N = \begin{cases} 1,3,5,\cdots \\ 2,4,6,\cdots \end{cases} \tag{43}$$

$$M_m = [(\lambda + 2\mu) j_0(k_m r) - 4\mu j_1(k_m r)/(k_m r)]. \tag{44}$$

2) Rectangular Pulse: We now can obtain the displacement and radial stress for a rectangular pulse of microwave energy by applying Duhamel's theorem [23] to the solutions expressed by (39) and (42). That is,

$$u(r,t) = \frac{\partial}{\partial t}\int_0^t F_i(t - t') u'(r,t')\, dt' \tag{45}$$

where $u'(r,t)$ is the solution given by (39) for the case of a sudden application of microwave radiation. An equivalent expression can, of course, be written for the radial stress. Therefore, by substituting (12) and (39) into (45), we have for the displacement

$$u(r,t) = u_0 Q t + \sum_{m=1}^{\infty} A_m j_1(k_m r) \frac{\sin \omega_m t}{\omega_m}, \qquad 0 \le t \le t_0 \tag{46}$$

$$u(r,t) = u_0 Q t_0 + \sum_{m=1}^{\infty} A_m j_1(k_m r)\left[\frac{\sin \omega_m t}{\omega_m} - \frac{\sin \omega_m (t - t_0)}{\omega_m}\right], \qquad t \ge t_0. \tag{47}$$